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Supra R*-Closed Sets and Supra R*-Continuity

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Abstract

The aim of this paper is to define and investigate supra R*-closed sets in topological space. Further, we discuss the concept of supra R*-continuity and supra R*-irresolutness .

Keywords: μR*-C(X), μR*-O(X), μR*-continuous, μR*-irresolute

Introduction

The study of generalized closed sets in topological spaces was initiated by Levine [7].The modified forms of generalized closed sets were studied by [1,2,9,10].C.Janaki and Renu Thomas introduced the R*-closed sets[4] .The notion of supra topological spaces was introduced by A.S.Mashhour [6] et al in 1983.

The purpose of this paper is to introduce and investigate a new class of sets called the supra R*-closed sets (μR*-C(X)) and study its relation with other supra closed sets. Further we study supra R*-continuous and supra R*-irresolute function and their properties.

Preliminaries

Definition 2.1:

Let X be a non empty set .The sub family μ $\subseteq P(X)$ where $P(X)$ is the power set of X, is said to be the supra topology on X if X and $\varphi \in \mu$ and μ is closed under arbitrary unions. The pair (X,μ) is called a supra topological space. The elements of μ are said to be supra open sets .The complement of supra open sets are supra closed sets.

Definition 2.2:

(i) The supra closure of A[8] denoted by $cl^{\mu}(A)$,is defined as $cl^{\mu}(A) = \cap\{B:B$ is a supra closed set and $A \subseteq B\}$

(ii) The supra interior of A[8] denoted by $int^{\mu}(A)$,is defined as $int^{\mu}(A) =\{B:B$ is a supra open set and $B \subseteq A\}$

Definition 2.3: Let (X,τ) be a topological space and μ be the supra topology on X.We call μ to be a supra topology associated with τ if $\tau \subset \mu$

Definition 2.4 A subset A of a topological space (X, τ) is called supra regular open [5] if $A = int^{\mu}(cl^{\mu}(A))$ and supra regular closed [5] if $A = cl^{\mu}(int^{\mu}(A))$.The family of all supra regular closed sets of X is denoted by SRO(X).If a set is both supra regular open and supra regular closed then it is supra regular clopen and denoted by μr-clopen.

Definition 2.5

1. The intersection of all supra regular closed subset of (X,τ) containing A is called the supra regular closure of A and is denoted by $rcl^{\mu}(A)$.
2. The supra regular interior of A is the union of all supra regular open sets of (X, τ) contained in A.

Definition 2.6 A subset A of a space (X,τ) is called supra regular semiopen if for every supra regular open set U, such that $U \subset A \subset cl^{\mu}(U)$. The family of all supra regular semi open sets of X is denoted by S-RSO(X).

Definition 2.7: Let (X,τ) and (Y,σ) be two topological spaces and $\tau \subseteq \mu$.

A function f: $(X,\mu) \rightarrow (Y,\sigma)$ is called

(i) supra continuous [3] if the inverse image of each open set of Y is a supra open set in X.

(ii) supra regular continuous [3] if $f^{-1}(V)$ is supra regular closed set in X for every closed set V in Y.

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Definition 2.8: Let (X, μ) and (Y, σ) be two topological spaces and $\tau \subseteq \mu$. and $\sigma \subseteq \lambda$

A function f: (X, μ) → (Y, λ) is called

(i) supra irresolute [3] if the inverse image of each supra open set of Y is a supra open set in X.

(ii) supra regular irresolute [11] if f^{-1} (V) is supra regular closed set in X for every supra regular closed set V in Y.

SUPRA R*-CLOSED SETS (μR*-CLOSED SETS)

Definition 3.1. A subset A of a supra topological space (X,μ) is called supra R*-closed if $rcl^{\mu}(A) \subset U$ whenever $A \subset U$ and U is supra regular semiopen in X. We denote the set of all supra R*- closed sets in X by μR*-C(X).

Result 3.2: Supra closed and supra R*-closed are independent.

Example 3.3: Let $X = \{a,b,c,d\}$

$$\mu = \{X,\phi,\{a\},\{b,c\},\{a,b,c\},\{a,b,d\}\}$$

$$\mu R^* - C(X) =$$

$$\{X,\phi,\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\},$$

Here $\{c\}$ is supra closed but not μR*-closed. Also thecollection

$\{\{a,b\},\{a,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,c,d\}\}$ is supra R*-closed but not supra closed.

Theorem 3.4: Finite union of supra R*-closed sets is supra R*-closed.

Proof: Let U be a supra regular semi open set in X . Also let A and B be supra R*-closed set in X. Hence $rcl^{\mu}(A) \subset U$ and $rcl^{\mu}(B) \subset U$,therefore $rcl^{\mu}(A) \cup rcl^{\mu}(B) \subset U$.

Implies $rcl^{\mu}(A \cup B) \subset U$. Hence the finite union of supra R*-closed sets is supra R*-closed.

Result 3.5: Every Supra regular closed set is supra R*-closed but not conversely.

Proof: Straight forward.

Example 3.6

Let $X = \{a,b,c,d\}$

$$\mu = \{X,\phi,\{a\},\{c,d\},\{a,c\},\{a,c,d\}\}$$

$$\mu R^* C(X) =$$

$$\left\{\begin{matrix} X,\varphi,\{b\},\{a,b\},\{a,c\},\{a,d\},\{b,c\}, \\ \{b,d\},\{a,b,c\}, \\ \{a,b,d\},\{a,c,d\},\{b,c,d\} \end{matrix}\right\},$$

$$\left\{\begin{matrix} \{b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\}, \\ \{a,b,d\},\{a,c,d\} \end{matrix}\right\}$$

are not supra regular closed .

Theorem 3.7: If a subset A of X is μR *-closed set in X , then $rcl^{\mu}(A) \backslash A$ does not contain any non-empty supra regular semiopen set in X.

Proof: Suppose that A is μR*-closed set in X . Let U be a supra regular semiopen set such that $rcl^{\mu}(A) \backslash A \supset U$ and $U \neq \phi$. Now $U \subset X \backslash A$ implies $A \subset X \backslash U$ and $A \subseteq U^{c}$.Since A is μR *-closed in X, $rcl^{\mu}(A) \subset U^{c}$.So $U \subset X \backslash rcl^{\mu}(A)$,hence $U \subset rcl^{\mu}(A) \cap X \backslash rcl^{\mu}(A) = \phi$.This shows that $U = \varphi$, which is a contradiction. Hence $rcl^{\mu}(A) \backslash A$ does not contain any non-empty supra regular semiopen set in X.

Theorem 3.8: For any element $x \in X$. The set $X \backslash \{x\}$ is supra R*-closed or supra regular semiopen.

Proof: Suppose $X \backslash \{x\}$ is not supra regular semi open, then X is the only supra regular semiopen set containing $X \backslash \{x\}$. This implies $rcl^{\mu}\{X \backslash \{x\}\} \subset X$. Hence $X \backslash \{x\}$ is supra R*-closed or supra regular semiopen set in X.

Theorem 3.9: If A is supra regular open and μR*-closed. Then A is supra regular closed and hence μr-clopen.

Proof: Suppose A is supra regular open and μR*-closed.$A \subset A$ and by hypothesis $rcl^{\mu}(A) \subset A$. Also $A \subset rcl^{\mu}(A)$, so $rcl^{\mu}(A) = A$. Therefore A is supra regular closed and hence μr-clopen.

Theorem 3.10 If A is an μR*- closed subset of X such that $A \subset B \subset rcl^{\mu}(A)$,then B is an μR*-closed set in X .

Proof: Let A be a μR*-closed set of X such that $A \subset B \subset rcl^{\mu}(A)$.Let U be a supra regular semiopen set of X such that $B \subset U$, then $A \subset U$. Since A is μR*-closed, we have $rcl^{\mu}(A) \subset U$. Now $rcl^{\mu}(B) \subset rcl^{\mu}(rcl^{\mu}(A)) = rcl^{\mu}(A) \subset U$, therefore B is an μR*-closed set in X.

Theorem 3.11: Let A be the μR*- closed in (X, τ). Then A is supra regular closed if and only if $rcl^{\mu}(A) \backslash A$ is supra regular semiopen.

Proof: Suppose A is supra regular closed in X. Then $rcl^{\mu}(A) = A$ and so $rcl^{\mu}(A) \backslash A = \phi$,which is supra regular semiopen in X.

Conversely, suppose $rcl^{\mu}(A) \backslash A$ is supra regular semiopen in X . Since A is μR*-closed by theorem 3.7 $rcl^{\mu}(A) \backslash A$ does not contain any non-empty supra

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regular semiopen set in X . Then rcl$^{\mu}$(A)\A $=\phi$ .Hence A is supra regular closed in X .

4. μR*-open sets

Theorem 4.1: A subset A of X is said to be supra R*-open if and only F$\subseteq$ rint$^{\mu}$(A) whenever F is supra regular semiopen and F$\subseteq$ A.

Proof**:** Necessity

Let F be supra regular semiopen such that F$\subseteq$ A. X-A $\subseteq$ X-F.Since X-A is μR*-closed

rcl$^{\mu}$(X-A)$\subseteq$X-F.rcl$^{\mu}$(X-A)=X-rint$^{\mu}$(A)$\subseteq$X-F. Thus F $\subseteq$ rint$^{\mu}$(A) .

Sufficiency

Let U be any supra regular semiopen set such that X - A$\subseteq$U and by hypothesis

X - U $\subseteq$ rint$^{\mu}$(A).Since rcl$^{\mu}$(X-A) = X- rint$^{\mu}$ A$\subseteq$ U .Therefore X-A is μR*-closed and hence A is μR*-open.

Theorem 4.2: Finite intersection of supra R*-open sets is supra R*-open.

Proof: Let A and B be μR*-open sets in X. Then

$A^c \cup B^c$ is μR*-closed set. This implies

$(A\cap B)^c$ is supra R*-closed set. Therefore A$\cap$B is supra R*-open.

Theorem 4.3: If A is μR*-closed subset of (X,τ) and F is supra regular closed set such that

F $\subseteq$ rcl$^{\mu}$(A) – A. Then F = ϕ and thus F $\subseteq$ rint$^{\mu}$(rcl$^{\mu}$(A) – A).

By Theorem 4.1, rcl$^{\mu}$(A) - A is μR*- open.

Remark 4.4

1. The intersection of two supra R*- closed set is generally not a supra R*- closed set.
2. The union of two supra R*- open set is generally not a supra R*- open set.

Example 4.5:

Let X={ a,b,c,d} τ={ X,φ,{a},{ c,d},{ a,c,d} μ={ X , φ,{a}, {a,c},{c,d},{a,c,d}}

μR*-C(X)={{a,c,d}, {a,d},{a,c}, {c,d},{a,b,c},{a,b,d},{a,c,d}, X ,φ}

Let A={ a,b,c} B={ a,b,d}

A$\cap$B={a,b} $\notin$ μR*-C(X)

μR*O(X)={{b},{c},{d},{a,b},{b,c},{b,d},{a,b,c},{a,b,d},{b,c,d}, X,φ}

A={c} B={d}

A$\cup$B= {c,d} $\notin$ μR*-O(X)

5. μR*- continuous and μR*-irresolute functions

Definition 5.1: A function f: (X, τ) → (Y,σ) is called μR*-continuous function if f^{-1}(V) is supra R*closed in (X,τ) for every closed set V in (Y,σ).

Definition 5.2: A function f: (X,τ) → (Y,σ) is called μR*-irresolute if f^{-1}(V) is μR*-closed in (X,τ) for every μR*-closed set V in (Y,σ).

Definition 5.3: A function f: (X,τ) → (Y,σ) is called supra regular continuous if f^{-1}(V) is supra regular closed in (X,τ) for every supra closed set V of (Y,σ).

Remark 5.4: The composition of two supra R*-continuous function need not be supra R*-continuous.

Example 5.5

X= Y= Z = {a,b,c,d}

μ={X, ϕ,{a},{d},{a,d},{a,b},{a,b,d}}

σ={X, ϕ,{a},{a,b},{a,b,c}}

η ={X, ϕ,{a},{c,d},{a,c,d}}.Define f : (X, τ)→ (Y,σ) by f (a) = b, f(b) =a, f(c)= d, f(d) =c and g : (Y,σ) →(Z,η) by is defined as the identity function .Here both f and g are supra R*-continuous but g$\circ$f is not supra R*-continuous. Since both $(g\circ f)^{-1}\{c,d\}=\{c,d\}\, and$ $(g\circ f)^{-1}\{a,c,d\}=\{a,c,d\}$ are not μR*-O(X).

Remark 5.6: μR*-irresolute function need not be supra R*- continuous and vice-versa.

Example 5.7:

Let $X = Y = \{a,b,c,d\}$

$\mu = \{X,\phi,\{a\},\{b\},\{a,b\}\}$

$\sigma = \{Y,\phi,\{a\},\{b\},\{a,b\},\{a,b,c\}\}$

μR*-C(X)=$\left\{\begin{matrix} X,\varphi,\{a,b\},\{c,d\},\{a,b,c\}, \\ \{a,c,d\},\{b,c,d\},\{a,b,d\} \end{matrix}\right\}$

μR*-C(Y) ==$\left\{\begin{matrix} Y,\varphi,\{a,b\},\{c,d\},\{a,b,c\}, \\ \{a,c,d\},\{b,c,d\},\{a,b,d\} \end{matrix}\right\}$

Define f: (X, μ) → (Y,σ) as the identity mapping, f^{-1}\{d} = {d} is not μR*-closed, hence not μR*-continuous but is μR*-iressolute.

Example 5.8: Let $X = \{a,b,c,d\}$

μ={ φ ,X,{d},{b,c}{b,c,d}{a,c,d}}

σ ={X, φ ,{b,c}{a,b,c}}

μR*-C(X)=

$\left\{\begin{matrix} X,\varphi,\{d\},\{a,d\},\{b,d\},\{c,d\},\{a,c,d\}, \\ \{b,c,d\},\{a,b,d\} \end{matrix}\right\}$

μR*-C(Y) = all subsets of Y.

Define f: (X,μ) → (Y,σ) as the identity mapping, f is μR*-continuous but is not μR*-irresolute.

Remark 5.9: Every supra regular continuous function is μR*-continuous but not conversely.

Let $X = Y = \{a,b,c,d\}$

$\mu = \{X,\phi,\{a\},\{d\},\{a,d\},\{a,b\},\{a,b,d\},\{a,c,d\}\}$

$\sigma = \{Y,\phi,\{a\}\}$

Define $f:(X,\mu)\to(Y,\sigma)$ as the identity mapping, f is μR*-continuous but not supra regular continuous.

Proposition 5.10: The composition of two supra R*-irresolute functions is supra irresolute.

Proof: Obvious

Proposition 5.11:

If f:(X,μ) → (Y,σ) then if f is supra R*-continuous, then $cl^{\mu}(f^{-1}(V)) \subset (f^{-1}(\overline{V}))$ for every V$\subset$Y.

Proof: Let V$\subset$Y. Then $\overline{V}$ is closed in Y, then $f^{-1}(\overline{V})$ is μR*-closed in X by the above result.. Therefore $f^{-1}(\overline{V}) = cl^{\mu}(f^{-1}(\overline{V}) \supset cl^{\mu}(f^{-1}(V)).$

Definition 5.12: A space (X, μ) is called supra R*-$T_{1/2}$ –space (μR*-$T_{1/2}$-space) if every supra R*-closed set is supra regular closed.

Theorem 5.13: Let f: (X,μ) → (Y,σ) and g: (Y,σ) → (Z,η) be any two function, then

1. g ∘ f is μR*-continuous if g is supra regular continuous and f is μR*-irresolute.
2. g ∘ f is μR*-continuous if is g is μR*-continuous and f is μR*-irresolute.
3. g ∘ f is-supra regular continuous if f is μR*-irresolute, g is μR*-continuous and X is a supra $T_{1/2}$ –space.

Proof:

(1).Let V be a supra closed set in (Z,η). Then g^{-1}(V) is supra regular closed set in (Y,σ). Since f is μR*-irresolute, $f^{-1}(g^{-1}(V))$ = $(g\circ f)^{-1}(V)$ is μR*-closed in (X,τ). That is $(g\circ f)^{-1}(V)$ is R*-closed in X. Hence g ∘ f is μR*continuous.

(2). Let V be supra closed set in (Z,η). Since g is μR*-continuous and g^{-1}(V) is μR*-closed set in (Y, σ). As f is μR*-irresolute, $f^{-1}(g^{-1}(V))$ = $(g\circ f)^{-1}(V)$ is μR*-closed in X. Hence g ∘ f is μR*- continuous.

(3). Let V be supra closed in (Z,η). Since g is μR*-continuous, g^{-1}(V) is μR*-closed in (Y,σ). As f is μR*-irresolute $f^{-1}(g^{-1}(V))$ = $(g\circ f)^{-1}(V)$ is supra R*-closed in X. Therefore g ∘ f is supra R*-continuous.Also since X is a supra $T_{1/2}$ space $(g\circ f)^{-1}(V)$ is supra regular closed and hence g ∘ f is supra regular continuous.

Theorem 5.14: For a topological space, the following conditions are equivalent.

1. X is supra R*-$T_{1/2}$-space.
2. Every singleton set in X is either regular semi closed or supra regular open.

Proof:(i)$\Rightarrow$(ii):Let $x \in X$ and assume {x} is not supra regular semi closed .Then X-{x}is not supra regular semi o pen and X-{x} is trivially a supra R*-$T_{1/2}$ closed space in which every supra R*-closed set is supra regular closed. $\Rightarrow$X-{x} is supra regular closed and {x}is supra regular open.

(ii) $\Rightarrow$(i):Let us assume every singleton of X is either supra regular semi closed or regular open. Let $A \subseteq X$ be supra R*-closed and $A \subseteq rcl^{\mu}(A)$ and let $x \in rcl^{\mu}(A)$.

To prove $rcl^{\mu}(A) \subseteq A$

Case (i): Let {x} be supra regular semi closed .Suppose {x} does not belong to A, then $\{x\} \subseteq rcl(A) - A$.But by corollary 3.19[10] $rcl(A) - A$ does not contain any non empty regular closed set in X. Hence{x}$\subseteq$A. $\Rightarrow$ $rcl^{\mu}(A) \subseteq A$.

The above implies $rcl^{\mu}(A) = A$ .Hence A is a supra regular closed set. Thus every supra R*-closed is supra regular closed and hence X is supra R*-$T_{1/2}$ –space.

Case (ii):Let{x}be supra regular open .Since {x} $\subseteq rcl^{\mu}(A)$, we have $\{x\} \cap A \neq \varphi$. Hence {x}$\subseteq A$.Therefore A is supra regular closed and hence every supra R*-closed set is supra regular closed.

Theorem 5.15:

1. SRO(X)$\subseteq$μR*-O(X)
2. A space X is supra R*-$T_{1/2}$-space iff SRO(X) = μR*-O(X)

Proof: (1) Let A be a supra regular open set in X. Then X-A is supra regular closed. Since every supra regular closed set is supra R*-closed$\Rightarrow$X-A is supra R*-closed. Hence A is supra R*-open which implies SRO(X)$\subseteq$μR*-O(X).

(2).Let X be a supra R*-$T_{1/2}$-space.Let A$\in$ μR*-O(X).Then X-A is supra R*-closed set. Since X is supra R*-$T_{1/2}$-space,X-A is supra regular closed. $\Rightarrow$ A is supra regular open in X. Hence SRO(X) = μR*-O(X).

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On the other hand, let SRO(X) = μR*-O(X).Let A be supra R*-closed. Then X-A is supra R*-open. X-A is regular open (by hypothesis).Hence A is supra regular closed and hence X is a supra R*-$T_{1/2}$-space.

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